Simple and Ordinary Factorial Cokriging

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ABSTRACT: This paper discusses factorial kriging from a viewpoint of linear system theory. Factorial kriging is presented as a parallel linear system of filters with a constraint on the output factorial components and a constraint on the linear system. For a stationary random function (RF), simple factorial cokriging (SFCK) may be shown to be equivalent to the Wiener filter. When ordinary cokriging is used to estimate factorial components, the procedure can be called ordinary factorial cokriging (OFCK), which may be considered as a generalization of the Wiener filter. Both SFCK and OFCK are developed in matrix forms. Subsequently, factorial filters can easily be calculated using the matrix solutions of kriging systems. Examples are provided to show applications of the methods as well as the calculation of factorial filters.

Keywords: simple factorial cokriging, ordinary factorial cokriging, Wiener filter, system theory, matrix formulation, block matrix inversion.

INTRODUCTION

It is well known that kriging is commonly used as an interpolation technique. Recent publications have shown an increase in the use of geostatistical filtering techniques. Chiles and Guillen (1984), Galli et al.(1984), and Zhang and Galli (1992) used kriging for geophysical data filtering. Sandjivy (1984) used kriging for geochemical data filtering. Ma and Royer (1987, 1988), and Conradsen and Nilsson (1987) developed kriging filters for remote sensing image processing. Daly et al. (1989) presented microscopic image filtering. Jaquet (1989) used kriging filtering techniques in petroleum exploration for mapping the thickness of a gas reservoir.

Different combinations of kriging were used for filtering. Some were variations of ordinary kriging with a particular combination of geometrical configuration (e.g., Conradsen and Nilsson filter). But almost all of the referenced works use factorial kriging in the filtering applications.

In this paper, factorial kriging is presented as a parallel linear system of filters. Factorial kriging has a constraint on the output components, and an additional constraint in the case of OFCK. Autokriging techniques estimate unknown points of a variable using available data of the same variable. Cokriging, on the other hand, estimates unknown points of a variable using data from the same variable and other related variables. Both autokriging and cokriging are interpola-

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tor. In contrast, factorial kriging decomposes an initial RF into several components, and the components need to be estimated by the initial RFs known values due to the lack of autokrigeability. From this point of view, factorial kriging can be viewed as a particular case of cokriging where the primary variable does not have any autokrigeability.

The spatial correlation modelling is not discussed in this paper. Instead, it is assumed that the phenomenon under study contains two or more underlying processes, which has been previously modeled by a nested covariance. Thus, based on the matrix formulation of autokriging (Ma, 1987) and cokriging (Myers, 1982), estimation of the filtered components for simple factorial cokriging is formulated in a matrix form. Since ordinary factorial cokriging is a constrained system, it will be formulated in a block matrix equation, and solved by a block matrix inversion formula. Hence, the weight vectors of factorial kriging are expressed as a function of the covariance and the constraint, and the results are clearly determined.

FACTORIAL KRIGING FROM A VIEWPOINT OF LINEAR SYSTEM THEORY

In linear system theory, if the input to the system (Fig.1) is a stochastic process $Z(x)$, then the resulting output $Y(x)$ is given by

$$Y(x) = \int_{-\infty}^{+\infty} Z(x-\alpha)\, L(\alpha)\, d\alpha \qquad (1)$$

which is also a stochastic process (Papoulis, 1977, p.305).

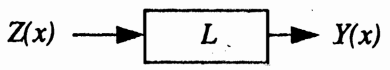

Fig.1 Linear System

The linear system L can also be interpreted as a linear filter in the sense that the output RF $Y(x)$ is a filtered process of the input RF $Z(x)$ (Roubine, 1970, p.117).

From a viewpoint of the linear system, factorial kriging is a multi-component filtering process. The input is a stochastic process $Z(x)$, whereas the output is p elementary stochastic processes $Y_i(x)$, as illustrated by Eq. (2) and Fig.2.

$$Z(x) = \sum_{i=1}^{p} Y_i(x) \qquad (2)$$

where the RFs $Z(x)$ and $Y_i(x)$ generally are not zero-mean processes.

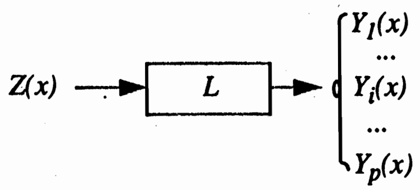

Fig. 2 Factorial Kriging

This multi-component filtering can be seen as a linear system with p parallel filters Λ_i, as shown by Fig.3.

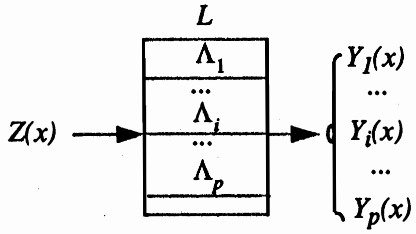

Fig. 3 Parallel System of Filters

Theoretically, the system L should have an infinite memory with the integral from $-\infty$ to ∞ as shown by Eq. (1) (Newton, 1989, p.86). In real applications, only finite-order systems can be realized. Thus, each resulting output RF $Y_i(x)$ can be characterized by the following discrete convolution equation with the order equal to N.

$$Y_i(x) = \sum_{\alpha=-N}^{N} Z(x-\alpha)\lambda_i(\alpha) \tag{3}$$

where the *i*th factorial component $Y_i(x)$ is filtered from the initial RF $Z(x)$ by the *i*th parallel filter of the linear system L.

In most applications, the initial RF $Z(x)$ does not have zero mean. The decomposition of the mean into different components is somewhat arbitrary (Matheron 1982, p. 5), but may be determined by the physical characteristics of the application. Without loss of generality, it is preferable to separate the mean in the decomposition model. Thus, Eq. (2) is rewritten as

$$Z(x) = \sum_{i=1}^{p} Y_i(x) + M(x) \tag{4}$$

where the input RF $Z(x)$ has the mean $M(x)$, which is not necessarily stationary, but the output components all have zero mean.

Eq. (4) is the general decomposition model of factorial kriging. When $M(x)$ is zero, the RF $Z(x)$ is a zero mean process. When the RF $Z(x)$ is stationary, $M(x)$ is a constant. When the RF $Z(x)$ is quasi-stationary (Papoulis, 1977, p. 302), $M(x)$ varies slowly. When the RF $Z(x)$ is intrinsic of order k, $M(x)$ is a drift.

With regard to the conventional linear system, factorial kriging can be seen as a filter with the following constraints:

<1> The output component RF $Y_i(x)$ is subject to the orthogonal condition

$$< Y_i(x), Y_j(x+h) > \;=\; C_{Y_iY_j}(h) \;=\; 0 \qquad for \;\; i \neq j \tag{5}$$

where $C_{Y_iY_j}(h)$ denotes the cross-covariance function between the components $Y_i(x)$ and $Y_j(x)$.

<2> If the input RF $Z(x)$ is not stationary, the linear system L should be constrained by the non-bias condition, which will be discussed in the *ordinary factorial cokriging* section.

The input RF $Z(x)$ to the system is sampled, whereas the factorial components are not directly sampled. Therefore, the components need to be estimated by cokriging, using data of the initial RF $Z(x)$. If the initial RF $Z(x)$ is second-order stationary, the system L is not subjected to the universal condition. Then, simple cokriging can be applied to estimate each output component $Y_i(x)$. This procedure can be called *simple factorial cokriging* (SFCK). If the initial RF $Z(x)$ is locally stationary, the linear system L should be constrained by the non-bias condition. Then, ordinary kriging can be applied to estimate each output factorial component $Y_i(x)$. The procedure can be called *ordinary factorial cokriging* (OFCK).

The estimation of each component $Y_i(x)$ reduces to finding the convolution function $\lambda_i(\alpha)$ of Eq. (3). In a discrete case, $\lambda_i(\alpha)$ is a weight vector. The weight vector can be obtained by solving either the SFCK or OFCK equation in matrix form. Then, the factorial filters can be easily calculated.

SIMPLE FACTORIAL COKRIGING

Consider a second-order stationary RF $Z(x)$. Since the mean m of RF $Z(x)$ is a constant value, Eq. (4) can be rewritten as:

$$Z(x) - m = \sum_{i=1}^{p} Y_i(x) \tag{6}$$

Because of the orthogonality between the different component RFs $Y_i(x)$ and $Y_j(x)$ given by Eq. (5), the following additive relationship of covariances is obtained:

$$C_Z(h) = \sum_{i=1}^{p} C_{Y_i}(h) \tag{7}$$

where $C_Z(h)$ is the auto-covariance function of the initial RF $Z(x)$, and $C_{Y_i}(h)$ is the auto-covariance function of the ith component RF $Y_i(x)$.

The ith stationary component RF $Y_i(x)$ can be estimated by the linear combination

$$Y^*_i(x) = \sum_{\alpha=1}^{n} \lambda_i^{\alpha} [Z(x_{\alpha}) - m] = {}_{sf}\Lambda_i^t Z_c \qquad for\ i = 1, \ldots, p \tag{8}$$

where Z_c is the zero-mean data vector of RF $Z(x)$, and ${}_{sf}\Lambda^t_i$ is the kriging weight vector (*where the superscript t denotes the transpose, and the left subscript sf indicates simple factorial cokriging*) as given in the following

$${}_{sf}\Lambda_i^t = [\lambda_{i,1}, \lambda_{i,2}, \ldots, \lambda_{i,n}] \qquad for\ i = 1, \ldots, p \tag{9}$$

$$Z_c = [Z(x_1), Z(x_2), \ldots, Z(x_n)]^t - mu = Z - mu \tag{10}$$

where u is the $(n \times 1)$ unit vector $u = [1, 1, \ldots, 1]^t$.

The variance of the estimation error ${}_{sf}\sigma_i^2$ is given by

$$\begin{aligned} {}_{sf}\sigma_i^2 &= E\{[Y_i(x) - Y_i^*(x)]\,[Y_i(x) - Y_i^*(x)]^t\} \\ &= E\{[Y_i(x) - {}_{sf}\Lambda_i^t Z_c]\,[Y_i(x) - {}_{sf}\Lambda_i^t Z_c]^t\} \\ &= E[Y_i^2(x) - 2\,Y_i(x)\,{}_{sf}\Lambda_i^t Z_c + {}_{sf}\Lambda_i^t Z_c Z_c^t\,{}_{sf}\Lambda_i] \\ &= C_{Y_i}(0) - 2\,{}_{sf}\Lambda_i^t C_{Y_iZ} + {}_{sf}\Lambda_i^t C_{ZZ}\,{}_{sf}\Lambda_i \end{aligned} \tag{11}$$

where $C_{Y_i}(0)$ is the variance of the ith component RF $Y_i(x)$, C_{Y_iZ} denotes the $(n \times 1)$ vector of cross-covariances between the component RF $Y_i(x)$ and the initial RF $Z(x)$, and C_{ZZ} denotes the $(n \times n)$ matrix of covariances between sample locations for the RF $Z(x)$ in the kriging neighborhood as illustrated by Eqs. (12) and (13)

$$C_{Y_iZ} = [C_{Y_iZ}(x - x_1), C_{Y_iZ}(x - x_2), \ldots, C_{Y_iZ}(x - x_n)]^t \tag{12}$$

$$C_{ZZ} = \begin{bmatrix} C_{zz}(x_1 - x_1) & C_{zz}(x_1 - x_2) & \ldots & C_{zz}(x_1 - x_n) \\ C_{zz}(x_2 - x_1) & C_{zz}(x_2 - x_2) & \ldots & C_{zz}(x_2 - x_n) \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ C_{zz}(x_n - x_1) & C_{zz}(x_n - x_2) & \ldots & C_{zz}(x_n - x_n) \end{bmatrix} \tag{13}$$

Minimizing the kriging variance of Eq.(11) yields

$$C_{ZZ}\,{}_{sf}\Lambda_i = C_{Y_iZ} \qquad for\ i = 1, \ldots, p \tag{14}$$

Because the different components $Y_i(x)$ and $Y_j(x)$ are orthogonal, the cross-covariance $C_{Y_iZ}(h)$ between the component $Y_i(x)$ and the initial RF $Z(x)$ is equal to the auto-covariance $C_{Y_i}(h)$ of the component $Y_i(x)$, as shown by the following development:

$$\begin{aligned} C_{Y_iZ}(h) &= E\{Y_i(x)\,[Z(x+h) - m_Z]\} \\ &= E\left[Y_i(x)\left(\sum_{j=1}^{p} Y_j(x+h) - m_Z\right)\right] = C_{Y_i}(h) \qquad for\ i = 1, \ldots, p \end{aligned} \tag{15}$$

Thus, Eq. (14) can be rewritten as

$$C_{ZZ}\,{}_{sk}\Lambda_i = C_{Y_i} \qquad for\ i = 1, \ldots, p \tag{16}$$

where C_{Y_i} is the vector of auto-covariances of the ith component $Y_i(x)$.

Equation (16) is the simple factorial cokriging system for the component $Y_i(x)$. The kriging weight vector, ${}_{sf}\Lambda_i$, can be obtained by pre-multiplying both sides by C_{ZZ}^{-1}. Then, the transpose of the resulting equation yields

$${}_{sf}\Lambda_i^t = C_{Y_i}^t C_{ZZ}^{-1} \qquad for\ i = 1, \ldots, p \tag{17}$$

Substituting Eqs. (15)-(17) into Eq. (11), the kriging variance of the ith component can be simplified to

$${}_{sf}\sigma_i^2 = C_{Y_i}(0) - C_{Y_i}^t C_{ZZ}^{-1} C_{Y_i} \tag{18}$$

Eq. (14) is the conventional normal equation for the Wiener filter. Simple factorial cokriging of Eq.(16) can be considered as the Wiener filter with an *a priori* orthogonal decomposition model given by Eq. (5). More generally, both the interpolation kriging (autokriging and cokriging), and the filtering kriging have the similar formalism to the Wiener filter. This similarity was first shown by Ma (1987), and later noted by several other authors (Ma and Royer, 1988a, 1988b; Olea, 1991, p. 71; Daly, 1991; and Deutsch and Journel, 1992, p. 62). In other words, simple kriging is formally a variation of the Wiener filter. Ordinary kriging and universal kriging are generalizations. One historical difference is that the Wiener filter was originally used for filtering, mainly in electrical engineering, whereas kriging has been essentially utilized for interpolation, especially in earth sciences. Another difference is that the Wiener filter was originally developed in one dimension, called time series, whereas kriging has been, from the beginning, formulated in three dimensions, but can be applied to one or two dimensional variables. In the following sections, by convention, the RFs $Z(x)$, $Y_i(x)$ denote one, two, and three dimensions, i.e. x can be considered as a vector.

ORDINARY FACTORIAL COKRIGING

When the RF $Z(x)$ is only locally stationary, its mean $m_Z(x)$ is dependent on the location. Then, the decomposition model (Eq. 4), the orthogonal condition (Eq. 5) and the additive relationship of covariances (Eq. 7) should be studied in a locally stationary kriging neighborhood, N_x.

The ith component RF $Y_i(x)$ can not be estimated by Eq. (8) unless the local mean has been previously estimated (see Ma, 1987, p.46-47; Ma and Royer, 1988a, p.21). An alternate method is to estimate a component $Y_i(x)$ by the initial RF $Z(x)$ with zero-mean constraint, as shown by

$$Y_i^*(x_0) = \sum_{i=1}^{n} {}_{of}\lambda_i \, Z(x_i) = {}_{of}\Lambda_i^t \, Z \tag{19}$$

$$\sum_{i=1}^{n} {}_{of}\lambda_i = 0 \quad or \quad {}_{of}\Lambda_i^t \, u = 0 \tag{20}$$

where ${}_{of}\Lambda^t_i$ is the kriging weight vector for the ith component RF $Y_i(x)$ (*where the left subscript of indicates ordinary factorial kriging*), and $\mathbf{Z}$ is the vector of the RF $Z(x)$'s known values in the stationary kriging neighborhood N_x

$$\begin{aligned} {}_{of}\Lambda_i^t &= \left[\lambda_{i,1}, \lambda_{i,2}, \ldots, \lambda_{i,n}\right] && for\ i = 1, \ldots, p \\ \mathbf{Z} &= \left[Z(x_1), Z(x_2), \ldots, Z(x_n)\right]^t && for\ x_\alpha \in N_x \end{aligned} \tag{21}$$

From Eq. (19), the estimation error ${}_{of}\sigma_i^2$ in the mean square can be written as

$$\begin{aligned} {}_{of}\sigma_i^2 &= \mathrm{E}[Y_i(x) - Y_i^*(x)]^2 = \mathrm{E}[Y_i(x) - {}_{of}\Lambda^t_i \mathbf{Z}]^2 \\ &= C_{Y_i}(0) - 2\,{}_{of}\Lambda_i^t \, \mathbf{C}_{Y_i} + {}_{of}\Lambda_i^t \, \mathbf{C}_{ZZ} \, {}_{of}\Lambda_i \end{aligned} \tag{22}$$

where $\mathbf{C}_{Y_i} = \mathbf{C}_{Y_i Z}$ from Eq.(5) is applied.

Based on Eqs. (20) and (22), the objective function is obtained

$$\phi({}_{of}\Lambda_i, {}_{of}\mu_i) = C(0) - 2\,{}_{of}\Lambda_i^t \mathbf{C}_{Y_i} + {}_{of}\Lambda_i^t \mathbf{C}_{ZZ} \, {}_{of}\Lambda_i + 2\,(\mathbf{u}^t \, {}_{of}\Lambda_i - 1)\, {}_{of}\mu_i \tag{23}$$

Minimizing ϕ with respect to the vector ${}_{of}\Lambda_i$ yields the following matrix equation

$$\mathbf{C}_{ZZ} \, {}_{of}\Lambda_i + \mathbf{u} \, {}_{of}\mu_i = \mathbf{C}_{Y_i} \tag{24}$$

and the constraint equation (20).

Combining Eqs. (20) and (24) yields the following block matrix equation

$$\begin{bmatrix} C_{ZZ} & u \\ u^t & 0 \end{bmatrix} \begin{bmatrix} {}_{of}\Lambda_i \\ {}_{of}\mu_i \end{bmatrix} = \begin{bmatrix} C_{Y_i} \\ 0 \end{bmatrix} \qquad for\ i = 1, \ldots, p \tag{25}$$

where C_{ZZ} is the covariance matrix of the RF $Z(x)$'s known values as defined by Eq. (13), ${}_{of}\mu_i$ is the Lagrange multiplier, and C_{Y_i} is the auto-covariance vector of the *ith* component RF $Y_i(x)$ as defined by Eq. (12).

Since the covariance matrix C_{ZZ} is positive definite, Eq. (25) can be further developed as follows:

$$\begin{bmatrix} {}_{of}\Lambda_i \\ {}_{of}\mu_i \end{bmatrix} = \begin{bmatrix} C_{ZZ} & u \\ u^t & 0 \end{bmatrix}^{-1} \begin{bmatrix} C_{Y_i} \\ 0 \end{bmatrix} = \begin{bmatrix} P & R \\ S & T \end{bmatrix} \begin{bmatrix} C_{Y_i} \\ 0 \end{bmatrix} \tag{26}$$

where the $(n \times n)$ matrix P, the $(n \times 1)$ vector R, the $(1 \times n)$ vector S, and the scalar T can be obtained by applying a block matrix inversion method (Ma, 1987, 1993)

$$T = -(u^t C_{ZZ}^{-1} u)^{-1}$$

$$R = C_{ZZ}^{-1} u\, (u^t C_{ZZ}^{-1} u)^{-1}$$

$$S = (u^t C_{ZZ}^{-1} u)^{-1} u^t C_{ZZ}^{-1}$$

$$P = C_{ZZ}^{-1} (I - u\, (u^t C_{ZZ}^{-1} u)^{-1} u^t C_{ZZ}^{-1}) \tag{27}$$

where I is the $(n \times n)$ identity matrix.

By substituting Eq. (27) into Eq. (26), the weight vector and the Lagrange multiplier are obtained:

$${}_{of}\Lambda_i^t = (P \cdot C_{Y_i})^t = C_{Y_i}^t C_{ZZ}^{-1} \left[I - (u^t C_{ZZ}^{-1} u)^{-1} u u^t C_{ZZ}^{-1} \right] \tag{28}$$

$${}_{of}\mu_i = S \cdot C_{Y_i} = (u^t C_{ZZ}^{-1}\ u)^{-1} u^t C_{ZZ}^{-1} C_{Y_i} \qquad for\ i = 1, \ldots, p$$

Pre-multiplying both sides of Eq. (24) by the vector ${}_{of}\Lambda_i^t$ yields

$${}_{of}\Lambda_i^t C_{ZZ}\, {}_{of}\Lambda_i + {}_{of}\Lambda_i^t\, u\, {}_{of}\mu_i = {}_{of}\Lambda_i^t C_{Y_i} \tag{29}$$

Then, by substituting the quantity ${}_{of}\Lambda_i^t C_{ZZ}\, {}_{of}\Lambda_i$ of Eq. (29) into Eq. (22), the kriging variance is simplified as follows:

$$
\begin{aligned}
{}_{of}\sigma_i^2 &= C_Y(0) - 2\,{}_{of}\Lambda_i^t\, C_{Y_i} + {}_{of}\Lambda_i^t\, C_{ZZ}\, {}_{of}\Lambda_i \\
&= C_{Y_i}(0) - {}_{of}\Lambda_i^t\, C_{Y_i} - {}_{of}\Lambda_i^t\, u\, {}_{of}\mu_i \\
&= C_{Y_i}(0) - {}_{of}\Lambda_i^t\, C_{Y_i} \qquad\qquad for\ i = 1, \ldots, p
\end{aligned} \tag{30}
$$

where the constraint ${}_{of}\Lambda_i^t\, u = 0$, provided by Eq. (20), is applied.

In the same way, the local mean $m_Z(x)$ can be estimated using the known values of the initial RF $Z(x)$ in the kriging neighborhood.

$$
m^*(x_0) = \sum_{\alpha=1}^{n} {}_{of}\lambda_m^{\alpha}\, Z(x_\alpha) = {}_{of}\Lambda_m^t\, Z \tag{31}
$$

$$
\sum_{\alpha=1}^{n} {}_{of}\lambda_m^{\alpha} = 1 \qquad or \qquad {}_{of}\Lambda_m^t\, u = 1 \tag{32}
$$

where Z is the vector of the RF $Z(x)$'s data as defined by Eq. (21), and ${}_{of}\Lambda_m^t$ is the weight vector for the local mean as defined by

$$
{}_{of}\Lambda_m^t = \left[\lambda_m^1\ \ \lambda_m^2\ \ \ldots\ \ \lambda_m^n\right]
$$

Using ordinary kriging for estimating the local mean leads to the following solution (Ma, 1993):

$$
\begin{aligned}
{}_{of}\Lambda_m^t &= (u^t C_{ZZ}^{-1} u)^{-1} u^t C_{ZZ}^{-1} \\
{}_{of}\mu_m &= -(u^t C_{ZZ}^{-1} u)^{-1}
\end{aligned} \tag{33}
$$

and the kriging variance for the local mean is given by

$$
\begin{aligned}
{}_{of}\sigma_m &= E\left[m(x) - m_Z^*(x)\right] = {}_{of}\Lambda_m^t\, C_{ZZ}\, {}_{of}\Lambda_m \\
&= -{}_{of}\mu_m = (u^t C_{ZZ}^{-1} u)^{-1}
\end{aligned} \tag{34}
$$

From Eqs. (28) and (33), it is seen that the kriging coefficients of the ith component RF $Y_i(x)$ are dependent on the auto-covariance functions of both the initial RF $Z(x)$ and the component RF $Y_i(x)$, whereas the kriging weight coefficients for the local mean are only dependent on the auto-covariance of the initial RF $Z(x)$.

EXAMPLES OF APPLICATIONS AND CALCULATIONS

Factorial kriging can be applied to filter out a component of a stochastic process. For instance, if the sample variogram is modeled by nested structures, the component corresponding to each of these structure can be filtered. Due to its mathematical generality, factorial kriging has been applied to geophysical and geochemical data interpretation, remote sensing image processing and petroleum exploration. Furthermore, it is also possible to utilize this method for post-processing of simulated data. Indeed, in some stochastic imaging, the simulated data have a higher nugget effect than the sample variogram (Deutsch and Journel, 1992). In such a case, it is possible to filter out the extra noise component using factorial kriging.

The following will show the calculation of both low-pass and high-pass filters using ordinary factorial kriging. As an illustration, the calculation is limited to a dichotomous decomposition

$$Z(x) = Y(x) + N(x) \tag{35}$$

where the local mean is assumed to be incorporated in the first component Y(x). In other words, the second component *N(x)* is a zero-mean process. Consequently, the filter of the component *Y(x)* is a combination of the equations (28) and (33):

$$_{of}\Lambda_Y^t = (u^t C_{ZZ}^{-1} u)^{-1} u^t C_{ZZ}^{-1} + C_Y^t C_{ZZ}^{-1}\left[I - (u^t C_{ZZ}^{-1} u)^{-1} u u^t C_{ZZ}^{-1}\right] \tag{36}$$

and the filter of the second component *N(x)* is given by

$$_{of}\Lambda_N^t = C_N^t C_{ZZ}^{-1}\left[I - (u^t C_{ZZ}^{-1} u)^{-1} u u^t C_{ZZ}^{-1}\right] \tag{37}$$

1. The variogram consists of a nugget effect plus an exponential model

$$\gamma(h) = \frac{2}{5} + \frac{3}{5} Exp(a = 12)$$

For this variogram, if a 2D (3 x 3) regular grid is used as the kriging neighborhood, the low-pass filter for estimating the component *Y(x)* is calculated by means of Eq.(36)

$$_{of}\Lambda_Y^t \approx \begin{bmatrix} 0.09 & 0.11 & 0.09 \\ 0.11 & 0.20 & 0.11 \\ 0.09 & 0.11 & 0.09 \end{bmatrix} \approx \frac{1}{10}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{38}$$

The high-pass filter for estimating the component $N(x)$ is calculated by means of Eq.(37)

$$ {}_{of}\Lambda^t_N \approx \begin{bmatrix} -0.09 & -0.11 & -0.09 \\ -0.11 & 0.80 & -0.11 \\ -0.09 & -0.11 & -0.09 \end{bmatrix} \approx \frac{1}{10}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \tag{39} $$

2. The variogram consists of a nugget effect plus a spherical model

$$ \gamma(h) = \frac{1}{7} + \frac{6}{7} Sph(a = 12) \qquad 0 < h < a $$

Similar to what was done with the previous model, the low-pass filter for estimating the component $Y(x)$ can be calculated using Eq.(36)

$$ {}_{of}\Lambda^t_Y \approx \begin{bmatrix} 0.05 & 0.1 & 0.05 \\ 0.1 & 0.4 & 0.1 \\ 0.05 & 0.1 & 0.05 \end{bmatrix} \approx \frac{1}{20}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 8 & 2 \\ 1 & 2 & 1 \end{bmatrix} \tag{40} $$

The high-pass filter for estimating the component $N(x)$ can be calculated using Eq.(37)

$$ {}_{of}\Lambda^t_N \approx \begin{bmatrix} -0.05 & -0.1 & -0.05 \\ -0.1 & 0.6 & -0.1 \\ -0.05 & -0.1 & -0.05 \end{bmatrix} \approx \frac{1}{20}\begin{bmatrix} -1 & -2 & -1 \\ -2 & 12 & -2 \\ -1 & -2 & -1 \end{bmatrix} \tag{41} $$

3. Pure nugget effect (a white noise process)

For a variogram with pure nugget effect, there is only the nugget component and the mean. However, since ordinary factorial cokriging is, in most cases, applied with a moving neighborhood, the mean is re-estimated for each kriging neighborhood. Hence, the mean can also be considered as a component. Thus, the dichotomous decomposition given by Eq. (35) reduces to

$$ Z(x) = M(x) + N(x) $$

The kriging weight vector of the mean component, which is a low-pass filter, is then directly calculated by Eq. (33) instead of Eq. (36). The high-pass filter is still calculated by Eq. (37). The results are, respectively,

$$ {}_{of}\Lambda^t_M = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{42} $$

and

$$ {}_{of}\Lambda'_N = \frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \tag{43} $$

The filter (42) is the traditional moving average, and the filter (43) is the 2D Laplacian filter. So, when the random process is not completely a white noise, neither the traditional moving average nor Laplacian filter is optimal in the sense of Wiener. Since factorial filters take into account the spatial correlation of data, they are statistically optimal. Indeed, by comparing the filters (38), (40) with the filter (42), it is easy to understand why the traditional moving average smooths too much for a non purely white noise random function.

CONCLUSIONS

Factorial kriging has been presented from a standpoint of linear system theory. Simple factorial cokriging can be considered as the Wiener filter with an *a priori* orthogonal decomposition model. Ordinary factorial cokriging can be viewed as a linear system with a constraint on the output components and a constraint on the filter.

Both SFCK and OFCK were developed in matrix form. By using the block matrix inversion method, the weight vector for OFCK was separated from the Lagrange parameter. Hence, when kriging is applied with a local moving neighborhood, the factorial filters can easily be calculated using the matrix solutions of SFCK and OFCK.

Factorial kriging can be considered as a particular case of multivariate kriging. This geostatistical filtering technique has been applied to diverse fields of the earth sciences. Due to its mathematical generality, it may have other potential applications, including non-stationary signal filtering and post-processing of stochastic simulations.

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